Qualitative Models in Conceptual Design: A Case Study

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Abstract Design is an important but little understood intelligent activity. *Conceptual* design is the transformation of functional specifications to an initial concept of an artifact that achieves them. Human designers rely heavily on the use of sketches, which can be thought of as qualitative models of a device. An appealing model of conceptual design is that of a mapping from *qualitative* functional specifications to a corresponding *qualitative* object model.
As a case study, I have investigated this model for the conceptual design of part shapes in elementary mechanisms, such as ratchets or gears. I present qualitative modelling formalisms for mechanical function, the *place vocabulary*, and for shape, the *metric diagram*. However, I show that qualitative functional attributes can not be mapped into corresponding qualitative attributes of a device that achieves them, and consequently that qualitative function can not be computed based solely on a qualitative object model. Only a significantly weaker functional model, *kinematic topology*, can be derived based on qualitative object models alone.
This result means that at least in mechanical design, sketches do not represent a *single* qualitative model, but must be interpreted as a *set* of possible precise models. Each step in the design process then refers to a particular precise model in this set. This novel interpretation of the use of sketches suggests alternatives to the popular model of conceptual design as a symbolic mapping of functional into object attributes.

ELEMENTARY MECHANISM DESIGN

Designing an artifact is a complex intellectual process of much interest to AI researchers. Most research in intelligent CAD systems has focussed on *detail* design, the adaption of an initial concept to precise specifications. Little is known about the process of *conceptual* design, the transition between functional specification and concept of an artifact that achieves it. As the artifact is only vaguely defined, conceptual design heavily involves qualitative reasoning and representations.

Figure 1: *Examples of higher kinematic pairs: a ratchet, a clock escapement, and gearwheels.*

I am investigating the problem of conceptual design for a specific and particularly intriguing sub-problem: the analysis and design of *higher kinematic pairs*, often called *elementary mechanisms*. A higher kinematic pair consists of two parts which achieve a kinematic function by interaction of their shapes. This class contains most of the "interresting" kinematic interactions, such as ratchets, escapements or gearwheels, as shown in the examples of Figure 1. Important properties of kinematic pairs are that each object has one degree of freedom (rotation or translation) only, and that their interaction can be modelled in two dimensions. The domain of kinematic pairs is ideal for studying design methodologies and representations because it is very rich in possible designs, but is also contained enough so that it can be studied in isolation without the need for extensive assumptions.

Design is a mapping of functional specifications to an actual physical device. A function of a mechanism is its behavior in a particular environment: a ratchet blocks a particular rotation, a Peaucellier device transforms circular motion of one point into straight line motion of another. A functional *specification* is a condition on a function of the device. A numerically precise specification of all functions of a device is often overly restrictive, and it would be pure coincidence if there actually existed a mechanism which satisfies them. In practice, the functional specifications are intentionally *vague*: they admit a whole range of numerical values for functional properties. In order to exploit the possibilities admitted by this vagueness, it must be represented in the functional specification, using a *qualitative* functional model. In this paper, I describe how *place vocabularies* ([Faltings, 1987a, Faltings, 1990]), a qualitative functional modelling language for elementary mechanisms, can form the basis for formulating qualitative functional specifications.

A common belief among designers is that the design process has a hierarchical structure: a very rough conceptual design is done first, and sucessively more and more details are filled in. For example, architects do their first sketches with a very coarse pencil so that they are not tempted to fill in too much detail. This suggests

Figure 2: *An appealing model of conceptual design: design specifications determine characteristics of a qualitative functional model, which are then mapped to attributes defining the qualitative object model. Note that the mapping can involve combinations of attributes.*

that at the stage of conceptual design, the design object should be modelled qualitatively. Conceptual design would then be a mapping from the qualitative functional model to the qualitative object model, as shown in Figure 2. This is the model of design implicitly or explicitly assumed by many researchers in intelligent CAD ([Yoshikawa, 1985, Tomiyama et al., 1989, Williams, 1990]). For exploring this model of conceptual design, I define a qualitative modelling formalism for shape, the *metric diagram*. The metric diagram is designed to represent the information contained in sketches used by human designers.

The most important problem in conceptual design is then the mapping between the functional and object models, i.e. between place vocabularies and metric diagram. It turns out that this mapping requires quite precise information (or assumptions) about the metric dimensions of the parts of the device. Only very weak functional representations, such as kinematic topology ([Faltings et al, 1989]), can be based on qualitative object models. I argue that this problem is not a result of the particular representations used in this paper, but is a general property of the domain of elementary mechanisms.

If precise metric dimensions are required in the design process, qualitative models of the artifact are not enough. As a result, I will argue that for most conceptual design problems, the model of Figure 2 should be replaced by the one in Figure 3. In this model, first-principles reasoning relates the *precise* dimensions of the object model to the qualitative functional characteristics of the device. The new model offers another, less obvious explanation of the fact that human designers like to use sketches: in conceptual design, the precise dimensions have to be changed very often, and a sketch can be reinterpreted as different (precise) models rather than having to be redrawn each time a dimension is changed! The results of my research provide strong evidence for this alternative interpretation.

Figure 3: *A more adequate model of the conceptual design process: attributes of a precise object model map to a qualitative functional model. Conceptual design involves frequent modifications of precise dimensions.*

What is a qualitative representation?

As there is no generally agreed on definition of what makes a model "qualitative," it is necessary to define the term for the purposes of this paper. The word "qualitative" is derived from "quality", which is a synonym of "property". Consequently, a qualitative model is composed of properties of the modelled domain, represented as predicates in first-order logic. More precisely, a qualitative model is

> *a model of a system in first-order predicate logic where symbols correspond to independent entities of the modelled system, and predicates correspond to connections between or properties of the symbols.*

This definition is consistent with all the major approaches to qualitative modelling, in particular it entails compositional and local models (as defined in [Bobrow, 1984]). Note, however, that models are usually represented using higher-level constructs, but could be translated into predicate calculus.

A first characteristic of models following this definition is that they are *compositional*: no symbol can refer to combinations of independent entities, so that the set of symbols in a composed system is simply the union of the symbols associated with each of its parts. This allows a model to be instantiated from a finite library of physical knowledge, as for example in Qualitative Process Theory ([Forbus, 1984]).

Of particular interest in this paper are the representations of quantities allowed in a qualitative model. The restriction that symbols and predicates must correspond to entities of the real world restricts the use of precise numbers to *landmark values*, fixed distinguished values for which such a correspondence can be established. It also rules out representations of quantity values by symbolic algebra, as this would require introducing predicates (such as multiplication) and individuals (subexpressions) which have no correspondence in the real world. The above definition of a qualitative model leaves only two ways of modelling the values of quantities:

- by a fixed set of qualitative values expressed as predicates on the quantity, such as POSITIVE(x), ZERO(x) or NEGATIVE(x).

- by relations between quantities or landmark values, such as GREATER(x,y).

The same applies to any individual which represents a continuously variable entity of the world, such as shapes. This is the definition of a "qualitative" model which I shall use throughout this paper.

QUALITATIVE REPRESENTATIONS OF FUNCTION

The model of conceptual design as a mapping from function to design object requires first of all a language for qualitatively specifying function. For many domains, the issue of what such a language should express is an open problem. For the limited domain of elementary mechanisms, however, a clear set of requirements can be stated. In the following, note that I use the word *function* to mean something distinct from the *purpose* of the device. As an example, consider a list of specifications that might lead to a ratchet device, shown in Figure 1:

- the device should permit continuous rotation of an axis in the counterclockwise direction, but mechanically block it in the clockwise direction.

- the permissible backlash in the clockwise direction is at most 0.25 rotations.

- the maximum torque required for turning in the counterclockwise direction is at most 3Nm.

The first element specifies a desired function as a list of required behaviors in two different environments[1]:

- given a torque on the shaft in one direction, the device should permit the rotation.

- given a torque in the opposite direction, the device should eventually produce a reaction force to it, and thus block the rotation.

The second specification is a kinematic restriction on the behavior of the device in response to a change in its environment, namely when the direction of the input torque is reversed from counterclockwise to clockwise. Likewise, the third specification imposes a condition on the numerical parameters of the behavior in an environment where there is both a torque and motion of the shaft in the counterclockwise direction. There do of course exist other restrictions, such as those on the size of the device, but those are not functional specifications.

The point of this example is to show that functional specifications in general take the form:

Environment $\Rightarrow$ Behavior, or
Environment $\Rightarrow$ Restriction on Behavior

[1] An automatic translation of the original form of specifications into this more complete form would require extensive knowledge about mechanisms and their use, and is not addressed in this paper.

where the $\Rightarrow$ often implies a causal connection, i.e. a property of the environment must produce the particular behavior.

We thus define a *function* as a pair of (Environment, Behavior), giving the behavior of the device in a particular environment. Note that many functions may never be specified: a pair of gearwheels may function as a conductor (or a space heater), but if it is never put in that environment this function is irrelevant. A functional *specification* is a condition on a function.

In almost all cases, the desired device has to function not only in particular, precisely specified environments, but in a whole range of them. Consequently, the environments in the functional specification are qualitative, most often given by ranges of parameter values. This means that the resulting behaviors, and the functional model as a whole, are also qualitative, although restrictions on behaviors could refer to precise numerical parameters.

The environment in which a mechanism is used can be specified as a set of qualitative values (or history of values) for the external parameters which influence the device. The types of qualitative values used in actual mechanism specifications are either:

- signs of parameters, e.g. "turns clockwise"
- intervals of parameters, e.g. "requires a force less than 3 N"

Specification of an admissible interval can be seen as specification of a sign with an added numerical restriction. For the sake of simplicity, I have limited this research to specifications which involve the sign only. Adding numerical restrictions requires additional modeling and can only increase the complexity of the models, so that this does not affect the main point of this paper.

For expressing qualitative behavior or restrictions on qualitative behavior in a qualitative functional model, a general modeling language for qualitative mechanical behavior is necessary. A good qualitative model of behavior is the *envisionment* ([De Kleer, 1977]) of the device. However, the envisionment itself is not appropriate for modeling function, as it cannot express the functional connection between the environmental inputs and the resulting behavior.

These functional connections are given as a set of inference rules or equations that relate individual parameters of the device. As an example, consider a contact between two objects whose positions along some axis (not parallel to the contact surface) are given by parameters a and b. The (unidirectional) properties of a the contact are captured by the following inference rules:

$$\frac{da}{dt} = + \quad \Rightarrow \quad \frac{db}{dt} = +$$
$$\frac{db}{dt} = - \quad \Rightarrow \quad \frac{da}{dt} = -$$

An increase in the parameter a results in an increase in parameter b (pushing), but decreasing a has no influence on b - one can not pull with the contact.

In general, a complex device has an internal state which defines the applicable functional connections between inputs and outputs. A formalism for representing mechanical function must represent the different states and possible transitions between them. A functional representation that fulfills these criteria is the *place vocabulary*

Figure 4: *The place vocabulary for the interaction of a pair of gear teeth. Note how it distinguishes places P_1 and P_2 where counterclockwise motion of gear 1 can push gear 2, places P_4 and P_5 where gear 2 can push gear 1 counterclockwise, and the slack state, P_3, where the two gears are not in contact. The superscripts indicate the periodic repetition of the interactions.*

([Faltings, 1987a, Faltings, 1990]). Each place[2] is characterized by a particular kind of object contact and the set of applicable qualitative inference rules between the dynamic parameters of the device. The places are arranged in a graph, whose edges define the possible transitions and are labelled with the conditions on qualitative motion under which the transitions can occur.

As an example, consider a typical place vocabulary for gearwheel interactions, shown in Figure 4. The set of places represents the qualitatively different kinds of object contacts and corresponding functional relations in the form of inference rules between motion parameters. Thus, the inference rules for P_1 and P_2 are different from those for P_4 and P_5, and P_3 has no attached inference rules at all. The place vocabulary provides a detailed model of the functions of a pair of gears, including the slack when the direction of rotation is changed.

Transitions between places depend on motions of the device. The adjacencies between places are marked with the qualitative derivatives of each of the motion parameters which might result in the given transition between places. This is an essential

[2]While "state" would be a better term, we use "place" for historical reasons, and to avoid confusion with kinematic states.

element of the detailed functional model, and necessary to compute an actual envisionment of behavior based on the place vocabulary. A detailed description of place vocabularies can be found in [Faltings, 1990].

The place vocabulary is a minimal formalism for functional modelling. Every change in the place vocabulary entails different states or inference rules which represent a potential difference in qualitative function of the device, so that all elements of a place vocabulary are *necessary* for the functional model[3] On the other hand, every change in qualitative function of the device must manifest itself as a variation in the place vocabulary, so that the place vocabulary is also a *sufficient* model of the qualitative function.

Realistic specifications almost never specify the complete functional model, but only part of it. The example of the ratchet specifications above refers only to forces on the input shaft, and says nothing about the movement of the pawl. Such specifications have to be completed with more detail before they can be used to define an actual device. This can be done either by instantiating a device from memory, or by using first principles to search for a satisfactory complete model. In both cases, a qualitative language for expressing complete functional models is required: either for representing and indexing the library of known functions, or for constructing a search space of functions. This task of *completing* the specifications is a very difficult part of design. In this paper I only address the representational issues involved.

A complete place vocabulary can then be matched to the functional specifications, either by

- envisioning the behavior in response to the environments of interest ([Nielsen, 1988]), computing a causal analysis and comparing it with the desired one, or
- using the behavioral rules to directly infer relations between input parameters and behavior, for example inferring the direction of motion of a gear in response to an input motion.

Place vocabularies thus provide a representation for modeling mechanical function and, consequently, functional specifications.

However, in some cases the place vocabulary shows more functional detail than is required by the specifications. For example, in many cases the slack in a pair of gearwheels is so small as to be negligible in the specifications and functional models. Several researchers have investigated the use of abstractions, either on the level of place vocabularies ([Nielsen, 1988]), or on the level of configuration spaces ([Joscowicz, 1989]). Such techniques can be used to construct abstracted place vocabularies which can be matched more efficiently to specifications.

MODELLING THE DESIGN OBJECT

When discussing concepts, people like to refer to *sketches*, which appear to be qualitative representations of some form. In fact, designers often insist on using an extra wide pen in order to make purposely rough sketches of their initial ideas. In this section, I

[3] Some place distinctions are due only to changes in object contacts. They could be abstracted away if this should not count as a functional distinction.

Figure 5: *Example of a sketch of a ratchet. Note that the sketch correctly models the discontinuities of the shapes (marked by circles) and the types of boundaries between them. However, the metric dimensions are much less precisely defined, leaving open the different interpretations shown on the right. This characteristic is shared with the metric diagram.*

develop a qualitative modelling formalism for shape which is designed to capture the information represented in a sketch.

In the domain of elementary mechanisms, a sketch is a rough drawing of the shapes of a mechanism which achieves a desired function. An example of such a sketch is shown in Figure 5. The sketch shows the example of a ratchet, a device which blocks clockwise rotation of the wheel (state shown in the sketch), but allows counterclockwise rotation. The most striking fact about the sketch is that the wheel as it is shown in the sketch can not even be turned a full rotation. Consequently, the device as shown in the drawing does not even achieve the function of a ratchet that the sketch is intended to demonstrate! Clearly, understanding of a sketch must involve an *interpretation* as a desired function, rather than just a simulation of the device as shown. A theory of how such an interpretation might be constructed is described later in this paper.

If a sketch can not be understood as a precise shape representation, what is the information it conveys? Note that a sketch correctly shows the discontinuities of the shape, as indicated by the circles in Figure 5, as well as the type of edges between discontinuities. The inaccuracies in metric dimensions leave open a range of possible interpretations, some of which are shown on the right in Figure 5. For elementary mechanisms, the information conveyed by a sketch consists of:

Figure 6: *The metric diagram for a ratchet mechanism. The object dimensions are defined as symbolic coordinates, which can then be represented by qualitative values. The coordinates of discontinuities are local to the particular object and defined in polar coordinates.*

- a graph of discontinuities and edges between them, and
- very approximate metric dimensions of the shape, and
- annotations, such as freedom of motion or periodicity of a shape.

This is the information modelled in the *metric diagram* of a shape, shown in the example in Figure 6. The structure of the shape is modelled as a graph of vertices and edges between them, indicated by the drawing of the shape in Figure 6. In my implementation, edges can be either straight lines or circular arcs.

Metric dimension parameters are associated with each of the elements of this structure. They are the positions of the discontinuities in polar coordinates, the positions of the centers of circular arc edges, and the distances of the objects in the frame of the mechanism. They can be represented in quantity spaces, systems of fixed intervals, precise numbers, or any other representation suitable to express the knowledge available about them. Annotations to the metric diagram express the freedom of motion of the parts, and the periodicity of the ratchet wheel.

The metric diagram is a qualitative representation: the graph modelling the connections can be expressed in predicate logic, and parameter values can be represented by a fixed set of qualitative values. I now show that there exists no simpler shape representation which is both sufficient to predict the different possible object contacts - an important precondition for any kinematic analysis - and is also qualitative according to the definition in the introduction. This claim follows from a proof that all of the elements of the metric diagram are required in any qualitative shape representation useful for qualitative kinematic analysis.

According to the definition of qualitative, each individual in a qualitative representation must correspond to an entity in the real world. Each part of a mechanism must therefore be modelled by distinct and independent individuals. The elements of the model of a single part shape can be justified as follows:

- **Discontinuities**: depending on the shape the part interacts with, each discontinuity can cause an isolated object contact to appear, so it must be an independent element of the model.
- **Edges**: different types of edges can result in different propagation of motion by the mechanism. Connections between edges and discontinuities define possible transitions between contacts. Edges and the connections they define must be another independent element of the model.
- **Positions of discontinuities**: changing the position of a single discontinuity can make an object contact possible or impossible, so each must be represented individually.
- **Distance between objects**: is required to predict object contact, and must be a separate quantity because of the independence of object models.
- **Annotations**: are either shorthand (periodicity), or express important information for kinematic analysis (freedom of motion).

I conclude that the metric diagram is a required part of any shape representation which is to be related to a model of mechanism kinematics.

Other types of parameterizations or decompositions can be imagined to decouple the structure from the dimensions of a shape, but they do not change the nature of the representation. The formalization by polar coordinates illustrated in Figure 6 is optimal in the sense that functional attributes depend as directly as possible on attributes of the representation. For example, the functional attribute that particular parts of two rotating shapes can touch depends only on the distance of these parts from the respective centers of rotation: a single parameter in each of the shape representations. For other attributes, the conditions are often more complicated, but the fact that only polar and cartesian coordinates are widely used by engineers makes it unlikely that there exists a different formalism to represent coordinates which results in more direct mappings.

An interresting fact about the metric diagram is that by changing the representation of quantities to real numbers, it can be turned into a precise object representation. This makes it a good framework for comparing qualitative predictions to precise calculations. In particular, the complexity of kinematic predictions can be compared on the basis of what accuracy of the values of metric diagram quantities is required to make the prediction.

MAPPING BETWEEN FUNCTION AND ARTIFACT

The purpose of modelling in design is to allow reasoning about the relationship between a designed object and its function. Ideally, this would be accomplished by a direct mapping between attributes of the functional model and attributes of the geometric

Figure 7: *Three different states of a ratchet which illustrate the three example attributes. Deciding whether the contact shown in A is possible amounts to linear distance comparisons. Determining the inference rule which holds in B requires evaluating a complex algebraic expression on dimensions. Showing that the transition between contacts shown in C is actually possible requires iterative numerical analysis.*

object model. A first difficulty here is that this is a one-to-many mapping: physical objects that achieve a particular function can be constructed in many different ways. The task of *identifying* (or adding) the object features intended to achieve a particular functional attribute is the responsibility of a particular design strategy. This paper adresses the issue of reasoning about what the features must look like to achieve the intended function.

Using the place vocabulary as a functional model, the following are examples of functional attributes which are of interest in design:

1. the feasibility of a particular object contact (place).

2. the inference rules which hold in a particular place.

3. the conditions for possible transitions between a pair of places.

Each of these attributes can be mapped into a condition on the model of the designed object, as shown in [Faltings, 1988c]. Figure 7 shows example positions of a ratchet which serve to illustrate the three types of attributes. The three examples illustrate a progression of complexity: for the first, the mapping can be based on a purely qualitative model, the second requires rather precise approximations of the metric dimensions, and the third example can in practice only be validated on a numerically precise model.

Mapping to a qualitative object model

For an intelligent CAD system based on a qualitative object model, it is important to be able to map the qualitative functional attributes into equivalent attributes of the object model. In this section, we show the attributes in a metric diagram which correspond to the functional attributes shown in the three examples of Figure 7.

Feasibility of an object contact: The functional attribute of the existance of a state where two particular object parts touch maps into an attribute on the relative dimensions of the objects. In particular, for two rotating objects such as in the ratchet example, two object parts with distance d_1 and d_2 from the respective centers can touch whenever

$$\begin{aligned} d_1 + d_2 &\geq d_{center} \text{ and} \\ |d_1 - d_2| &\leq d_{center} \end{aligned}$$

where d_{center} is the distance between the centers of rotation of the two objects.

Situation A in Figure 7 is representative of the touch between the tip of the lever and the side of the wheel's tooth. The place corresponding to this situation exists whenver the tip of the lever can touch *some* point on the edge, so that (using the notation of Figure 6 given earlier) the attribute of the metric diagram becomes:

$$\begin{aligned} min(d_5, d_6) + d_2 &\geq d_{center} \text{ and} \\ |max(d_5, d_6) - d_2| &\leq d_{center} \end{aligned}$$

The mapping of this functional predicate into the object model is thus a linear distance comparison. If the distance between the centers of rotation, d_{center}, is kept constant in the design problem, it can be expressed as an attribute of the *relative* values of the object dimensions. As these could be expressed qualitatively in a quantity space ([Forbus, 1984]), a purely qualitative metric diagram is sufficient as an object model which allows one to express this attribute. However, if the distance between centers of rotation becomes variable, the quantity space representation becomes insufficient. Note also that the chosen metric diagram is optimal for this case: it only involves a single parameter for each object.

Inference rules for a given place: In the situation B in Figure 7, it might be of interest to know in which direction the pawl will turn when the wheel is turned counterclockwise. This depends on the direction of the edge of the lever with respect to the dashed circle which describes the incremental freedom of motion of the wheel. If the edge "points" to the inside, as shown in the figure, the lever will turn clockwise, otherwise, it will turn counterclockwise. The attribute can be generalized from a single contact point to all configurations with the given contact, but for one contact, there may be up to three regions with different inference rules. The functional attributes are not single inference rules, but the different combinations of regions with different inference rules which can exist for the type of contact.

The condition is equivalent to the sign of a linear expression of the sines and cosines of the rotation angles in the configuration where the points of interest touch. However, these angles in turn depend on a nonlinear combination of object dimensions ([Faltings, 1987b]). For the example B shown in Figure 7, the inference rule to be applied in the place containing the configuration where the tip of the lever touches a tip of the wheel depends on the sign of (using the notation of Figure 6:

$$\sqrt{d_2^2 + d_3^2 - 2d_2d_3cos(\xi_2 - \xi_3)}\left(d_2^2 + d_6^2 - d_{center}^2\right) - 2d_2d_3d_6$$

This attribute is dependent on nonlinear combinations of distinct parameters of the metric diagram, which furthermore belong to independent objects. Since qualitative attributes can not depend on combinations of independent objects, it is not possible to store its value as part of a qualitative object model, but it must be composed from individual qualitative representations. Only symbolic algebra is sufficient to do this, and consequently symbolic algebra is also required to express the attributes corresponding to rules for force and motion propagation in the metric diagram. As any qualitative shape representation must contain the metric diagram, and the power to express such attributes makes the metric diagram non-qualitative, *there is no qualitative shape representation that would allow one to express this attribute.*

Conditions for place transition: Situation C in Figure 7 shows a configuration where the lever and wheel touch in two different points. This instantaneous situation represents a transition between two places, called a *subsumption*. The attribute which states the possibility of this direct transition, i.e. the existance of this subsumption configuration, is an important element of the functional model of the device.

If we attempt to map this attribute to the object model, represented by a metric diagram, it amounts to the existance of a configuration which simultaneously satisfies two nonlinear constraint equations. Using symbolic algebra, it is possible to derive an equivalent condition as the existance of a root of a six-degree polynomial. By applying algebraic decision methods ([Ben-Or et al., 1986]), it is possible to reduce this to a complex combination of algebraic predicates which express the condition, but these are highly nonlinear in the parameters of the metric diagram - so complex that it is impossible to print them readably on a single page. Besides the fact that the nonlinear combination of parameters of independent objects violates the condition of compositionality, the expression of the subsumption condition is much too complicated to be effectively used for reasoning.

The example of subsumptions points to even deeper problems with the mapping between qualitative models of function and objects. It is due to the fact that qualitative representations are *local*: all relations between individual symbols are defined and reasoned about individually. Consequently, in a qualitative analysis of kinematics, each object contact is reasoned about individually. This fails to take into account *interference* between object contacts: a particular state may in reality be impossible because it would create an overlap of other, not directly related parts of the mechanism. Such interference can be reliably inferred only from the presence of subsumption configurations - but these attributes in turn can not be formalized in a qualitative object model. I conclude from these arguments that qualitative object models are almost useless for making even qualitative predictions about function. However, they may have a limited usefulness in design for controlling search processes, as indicated later in the paper.

Mapping to a precise object representation

Even though most functional attributes can not be mapped directly into attributes on a *qualitative* object representation, they do define attributes of a *precise* object representation which can be reasoned about. Given a numerically precise model of the designed objects, its place vocabulary as a representation of the qualitative function can be computed using the methods described in [Faltings, 1990, Faltings, 1987b]). Each of the attributes of the place vocabulary can be labelled with the conditions on the object representation which are necessary to maintain its existance ([Faltings, 1988c, Faltings, 1988a]). For the existance of places or inference rules associated with them, these are the algebraic conditions on the object dimensions, as shown in the examples given earlier.

For reasoning about the existance of a subsumption, it is now sufficient to express the condition for maintaining the *particular way* in which the subsumption is achieved by the object shapes, not a condition for the existance of the subsumption in general. For this reason, it turns out to be possible to formulate the conditions for maintaining or achieving a particular subsumption in closed form ([Faltings, 1988a]). Even subsumptions can be reasoned about if a precise object model is used.

The many advantages offered by precise object representations leave open the question of why designers prefer rough sketches to precise drawings at the stage of conceptual design. I discuss possible explanations in the next section.

INTERPRETATION OF SKETCHES: KINEMATIC TOPOLOGY

The results of the preceeding sections leave open the question why human designers often insist on using sketches. There are two possible interpretations of this phenomenon: either the sketch represents a single *qualitative* model, or the sketch is a representation of a family of *precise* models.

The sketch as a single qualitative model

As has been shown by the preceeding discussion, interpreting a sketch as a single qualitative model can not be powerful enough to infer qualitative kinematic behavior. However, it turns out that a qualitative metric diagram - equivalent to a sketch - is sufficient to predict the *kinematic topology* ([Faltings et al, 1989]) of the device. Kinematic topology expresses the topology of the device's *configuration space*, the space spanned by the position parameters of the mechanism's parts. For many devices, the topology of its configuration space already says a lot about its function. For example, in a pair of gearwheels, the fact that the two gears can only move in coordination can already be deduced from the fact that the configuration space consists of several doubly-connected regions which "wrap around" both dimensions of configuration space ([Faltings et al, 1989]). On the other hand, topology is too weak for a qualitative simulation of the meshing of the gear's teeth, or the blocking behavior in a ratchet.

The computation of kinematic topology is most easily explained by reformulating the metric diagram as a decomposition into adjacent shape primitives, of which there are two types: *pieces* for convex sections and *cavities* for concave sections, as shown in the example of Figure 8. Note that this primitive decomposition is very similar to the discontinuity-based representation in the metric diagram, with the addition of the distinction between convex and concave discontinuities.

The interaction of a pair of shape primitives generates topological primitives. As shown (intuitively) in Figure 9, an interaction between two pieces generates a potential area of illegal configurations, called *obstacle*, and an interaction between a piece and a cavity generates a potential area of legal configurations, called *bubble*. Initial

Figure 8: *Example of the representation of a shape by primitives.*

connections between these topological primitives are given by the adjacencies of the shape primitives on the objects themselves. However, for determining whether primitives actually exist, and whether adjacent primitives intersect or not, it is necessary to know whether the extremal points of these primitives can touch. This is the same condition for possible object contacts which has already been discussed earlier, and can be expressed in a qualitative model.

However, even kinematic topology depends crucially on the existance or absence of global subsumptions, which establish additional connections between topological primitives and can have a profound effect on configuration space topology. In spite of this problem, kinematic topology and the associated primitive-based representation of object shape are useful for conceptual design. Because of the high degree of abstraction, the amount of ambiguity which results when subsumption conditions can not be evaluated is manageably low. For example, for a pair of gearwheels described in the primitive decomposition, there are only five different topologies to be considered ([Faltings et al, 1989]). With only approximate metric information, such as that provided by a sketch, the analysis of kinematic topology already allows us to predict that the gears either mesh or jam - a prediction which rules out many other forms of behavior and provides a focus for subsequent detail design. An analysis at this level also explains how people can pick out the desired function out of the many functions permitted by the inaccuracies of a sketch. Furthermore, as shown in [Faltings et al, 1989]), kinematic topology can be computed for any shape which can be decomposed into segments of convex and concave curvature. To my knowledge, it is the only form of kinematic analysis which does not require a precise representation of object shapes.

The sketch as a family of precise models

An important characteristic of a sketch is that its precise dimensions often do not represent a correctly functioning device. The sketch requires an *interpretation* as a device with different precise dimensions in order to support the desired explanations. More precisely, the sketch defines a metric diagram in which the parameter values are

Figure 9: *Interaction between two object pieces creates illegal configurations, interaction between piece and cavity allows legal configurations.*

underdetermined. This vagueness means that a single sketch can be interpreted as any of a family of possible precise models. In conceptual design, the sketch thus allows the designer to make frequent mental changes to his design without having to change the drawing - an important economy when such changes are frequent.

The most likely explanation of the designer's use of sketches is a combination of the two possibilities. On the one hand, the sketch itself defines a restricted domain in which a precise solution is searched. This is based on a single interpretation, for example based on kinematic topology. On the other hand, it allows the designer to re-use the same drawing throughout the frequent changes inherent in conceptual design. The arguments in this paper have shown that the interpretation of sketches as precise models is inevitable for design, and consequently that the popular model of design being based on mapping functional attributes to a single qualitative model represented in a sketch is wrong.

CONCLUSIONS

I started this paper with the hypothesis that conceptual design is a mapping from a qualitative model of function to a corresponding qualitative model of the designed artifact, a common model among researchers in intelligent CAD. This model of design, motivated by observation of human designers, is corroborated by early work on automatic circuit design ([De Kleer and Sussman, 1989]), but its application to other domains such as mechanical design has in fact never been investigated. This was the starting point for the case study presented in this paper.

I have shown how qualitative models of kinematic function and of object shapes can be constructed for the limited domain of this case study. However, it has proven impossible to generate useful direct correspondences between the qualitative functional models and the qualitative shape models. Even though this result is limited to a narrow domain, it shows that the hypothesized model of conceptual design can at least not be generally applicable.

This result suggests an alternative model of conceptual design: the iterative refinement of a *precise* model of the design object ([Faltings, 1988b]). In this approach, shape modifications are obtained by reasoning about the limits of validity of the functional attributes of a current design. This avoids the difficulties with qualitative object models, while maintaining the use of a qualitative functional model which can easily be related to specifications. A precise object model is also used in the work of Joskowicz and Addanki ([Joscowicz and Addanki, 1988]), who present an incomplete algorithm for mapping exact functional specifications into corresponding exact object shapes.

Why, then, do human designers like to use sketches? One reason is that a representation like kinematic topology is useful for controlling search: a design whose kinematic topology does not correspond to the desired one should not be pursued any further. But another, more important reason may be that while the sketch defines the correct metric diagram, its precise dimensions can be freely reinterpreted according to the interests of the analysis. The sketch thus allows the designer to make frequent mental changes to his design without having to change the drawing - an important economy when changes are frequent. Because of the difficulties with kinematic predictions discussed earlier, this latter reason seems much more plausible than the use of a sketch as a qualitative shape representation. It is also likely that this model of the designer's use of sketches also holds for other domains, such as architecture.

On a larger perspective, the results in this paper cast some doubt on hopes for useful qualitative object representations. This doubt is confirmed by a survey of the research results that have been achieved in qualitative physics. Of the 55 papers in a recent representative collection of papers dealing with qualitative physics ([Weld and De Kleer, 1989]), only three put an emphasis on the qualitative representation of *objects*, and these only in the context of functional predictions. All successful research in qualitative physics is primarily motivated by qualitative models of behavior and function, which in some cases can be mapped successfully to qualitative object models. The results of this case study may thus indicate a deeper truth about qualitative physics: qualitative function does not always correspond to qualitative object attributes!

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