Mechanical Engineering is More than Differential Equations

Boi Faltings
Swiss Federal Institute of Technology (EPFL)
1015 Lausanne, Switzerland

June 15, 1991

In their paper, "Prolegomena to Any Future Qualitative Physics," Elisha Sacks and Jon Doyle raise interesting points regarding the applicability of qualitative reasoning to solving practical engineering problems. They criticize in particular a line of research which they identify by the letters SPQR: the qualitative simulation of processes described by a set of differential equations. While many of their criticisms are interesting, the underlying assumption that all physical systems can be adequately represented by systems of equations is invalid.

Engineering has no doubt benefited strongly from using mathematical concepts such as differential equations and dynamical systems theory. However, an engineer who wants to apply these concepts first has to construct a model where their use is tractable. Consider as an example the design and analysis of mechanisms such as clockworks. An important mechanism is the ratchet shown in Figure 1, a device which allows rotation of the wheel in one direction only.

The literature gives many techniques for expressing the kinematic constraints of mechanisms as a system of equations. However, these techniques do not apply to the analysis of a ratchet mechanism. First, the kinematic constraints in a ratchet prohibit the parts from overlapping, but not from separating. They are *nonholonomic* ([NIK88]) constraints, expressed as *inequalities* rather than equalities. Systems of inequalities by themselves generally do not give the unique solutions required for numerical methods. Thus,

Figure 1: *A ratchet is a device which allows rotation of a wheel in one direction only. Its behavior can not be modelled by equations alone.*

for example, it is impossible to predict which of the two configurations shown in Figure 1 c) will follow the configuration in b). A numerical analysis of the kinematics is possible only when additional precise assumptions about external influences on the device are made, for which sufficient information is usually not available.

More seriously, the set of applicable constraints changes discontinuously as the mechanism moves: the constraints which apply in Figure 1 a) are very different from those which apply in b). Before kinematic constraints can be used, a *modelling* step must identify the conditions under which each constraint applies. For a complete analysis of the device's behavior, it will further be necessary to identify which combinations of applicable constraints exist and what transitions between them are possible. It is not clear how to carry out this modelling using general numerical techniques.

Both problems can be readily dealt with using a *qualitative* model of the ratchet's behavior. A qualitative model of the ratchet's behavior as a *place vocabulary* ([FAL90a]) gives us the following advantages:

- ambiguities introduced by nonholonomic constraints are accomodated in the qualitative model. It has been shown that place vocabularies allow qualitative simulations of complex devices such as clockworks ([NIE88]). Such a qualitative simulation is the only general analysis of purely kinematic behavior which is known today.

- place vocabularies explicitly identify different applicable constraint sets and possible transitions between them, thus providing substrates where numerical analyses become possible. Besides simulation, it is also possible to formulate knowledge about mechanical *technology*, such as the heat generated by friction of certain contacts or the maximum force which can be supported by a certain type of material.

The place vocabulary theory follows the general SPQR paradigm and solves an important open problem, that of general analysis of mechanism kinematics.

Another area where purely equation-based models are insufficient is the *design* of mechanisms. In design, a common problem is to change the shape of mechanism parts in order to correct its behavior. This requires that features of the devices *behavior* must be linked to features of its *structure*. General techniques such as sensitivity analysis allow this link to be derived from an abstract, equation-based model. However, they are only valid for *small* variations. Dealing with large structural variations requires a detailed analysis of the *relation* between behavioral and structural features, such as shown (for mechanical devices) by the author in [FAL87, FAL88]. The abstraction into a general formalism, the main feature of equation-based models, is not appropriate for solving this problem.

Note furthermore that the features used to specify mechanism behavior for design usually (i) are qualitative, and (ii) refer to particular states or sequences of states of the device: "... oscillates between a fast stroke to the right and a slow stroke to the left...", "... reaches a state where counterclockwise rotation is blocked...". The state-based SPQR formalism which Sacks and Doyle criticize is exactly what is required to formulate such features, and is in fact already applied in several studies of mechanical design ([JOAD88, FAL90b]).

In summary, there are many domains, such as mechanical engineering, where models based only on differential equations are not very useful. I have argued that

- qualitative models can deal with nonholonomic constraints and thus provide more adequate models of kinematic processes.
- qualitative models are the basis for numerical models of kinematic behavior as well as mechanical technology, and
- qualitative models of kinematics are necessary for mechanical design.

I have restricted my reply to mechanism examples in order to show that SPQR is indeed useful to engineers. More convincing arguments can be made in domains such as robotics or computer vision, where the need for qualitative reasoning is increasingly recognized.

Sacks and Doyle do make the point that modelling is a promising research topic for qualitative physics, and this view is supported by the arguments given here. However, I hope to have shown that the SPQR methodology they criticize is often the correct framework for formulating these models. Dynamical systems theory is useful for many engineering problems, but it is not sufficient for mechanism kinematics.

Acknowledgement

I would like to thank Dr. Ian Smith, Dept. of Civil Engineering, EPFL, for many discussions about qualitative physics in engineering and comments on drafts of this paper.

References

[FAL87] **Boi Faltings**: "Qualitative Kinematics in Mechanisms," Ph. D. Thesis, University of Illinois, 1987

[FAL88] **Boi Faltings**: "A Symbolic Approach to Qualitative Kinematics," *Proceedings of the 3rd International Conference on 5th Generation Computer Systems*, Tokyo, 1988

[FAL90a] **Boi Faltings**: "Qualitative Kinematics in Mechanisms," *Artificial Intelligence* **44**(1), June 1990

[FAL90b] **Boi Faltings**: "Qualitative Kinematics and Intelligent CAD," in: H. Yoshikawa, T. Holden (eds.): *Intelligent CAD, II*, North Holland, 1990

[JOAD88] **Leo Joscowicz, Sanjay Addanki**: "From Kinematics to Shape: An Approach to Innovative Design," *Proceedings of the National Conference on Artificial Intelligence*, St.-Paul, 1988

[NIE88] **Paul Nielsen**: "A Qualitative Approach to Rigid Body Mechanics," Ph. D. Thesis, University of Illinois, 1988

[NIK88] **Parviz E. Nikravesch**: "Computer-aided Analysis of Mechanical Systems," Prentice Hall, 1988